The Fearful Asymmetry of the Tyger's Stripes:

Pattern Formation in Nature

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This project explores how naturally formed patterns can be described using concepts found elsewhere in physics, such as symmetry breaking and non-equilibrium thermodynamics. It is found that many systems can be described using reaction-diffusion equations, and some simple models are simulated to show how patterns can be formed this way.

1. History

The concept of patterns spontaneously forming due to simple laws is a relatively new area of research in physics, and it could be a controversial one – it implies that symmetry breaking is required to make a system more ordered, and this would create the appearance of decreasing entropy. However, this is possible if the system is not in equilibrium; with a continual flow of energy in (and out) of the system, spontaneous pattern formations can be sustained. There are many examples of this in nature, such as Rayleigh-Bernard convection cells. Sometime in the 1950s, the Russian scientist Belousov discovered that certain chemical reactions continually "oscillated" between two different colours, with no other changes to the system. This was considered impossible by scientists at the time, but is now understood to be a non-equilibrium reaction-diffusion system. Alan Turing attributed reaction-diffusion systems to some mechanisms in biology[1], and subsequent research has shown that many more areas of science could potentially be explained with this approach.

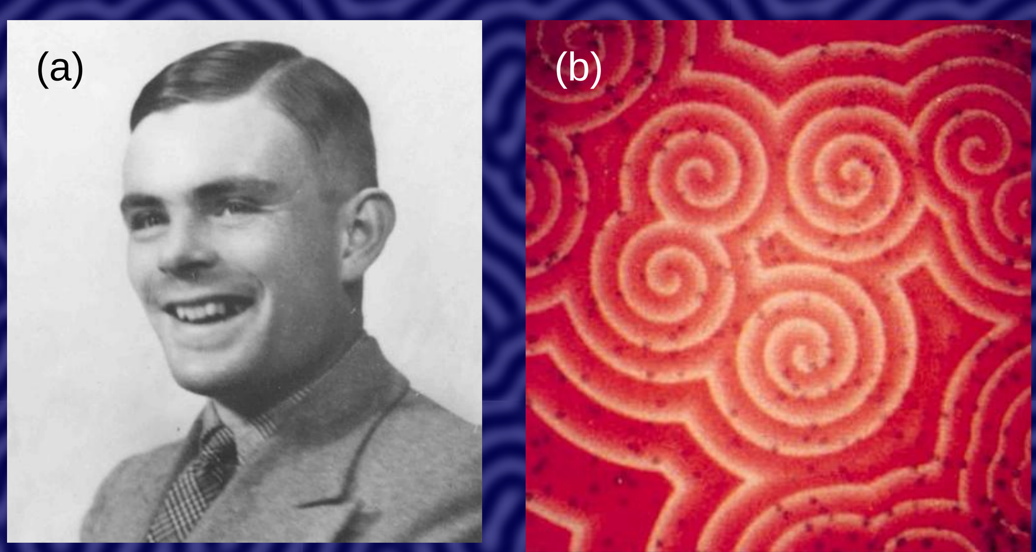

Figure 1. (a) A portrait of Alan Turing[i] who researched the role of reaction-diffusion systems in biology. (b) A Belousov-Zahbotinsky reaction[ii] – the first reaction-diffusion system to be recreated in the laboratory.

2. Reaction-Diffusion Systems: the Gray Scott Model

One of the simplest models for a reaction diffusion system is the Gray-Scott model[2]. This models the reaction

$$U + 2V \rightarrow 3V \qquad (1)$$

where *U* and *V* are chemical reactants. The equations that describe the system are

$$\frac{\partial v}{\partial t} = D_v \nabla^2 v - Auv^2 - fv - kv \qquad (2)$$

$$\frac{\partial u}{\partial t} = \underbrace{D_u \nabla^2 u}_{\text{Diffusion}} \underbrace{- Auv^2 + f - fu}_{\text{Reaction}} \qquad (3)$$

These describe the chemicals as they diffuse through the system at different rates, and react with each other as in (1). The chemical reaction has an important feature that allows it to behave the way it does – it produces one of its reactants, and this makes it autocatalytic. It is this feedback loop that produces the pattern forming behaviour.

The Gray-Scott model can be simulated using a cellular automaton, by introducing a 2D slab discretised into many cells. Within each cell, there is a concentration of *U* and *V* whose behaviour depends on the concentrations of its neighbouring cells. The differential equations (2) and (3) can be rewritten discretely so it is possible to solve them for multiple time steps. Most of the project was involved in producing simple cellular automata (such as the Game of Life) and learning how to discretise certain physical situations, for example a wave on a string or diffusion of heat through a slab. Modelling this produces many interesting patterns, that can be altered by adjusting the reaction/diffusion parameters (see Fig. 2.)

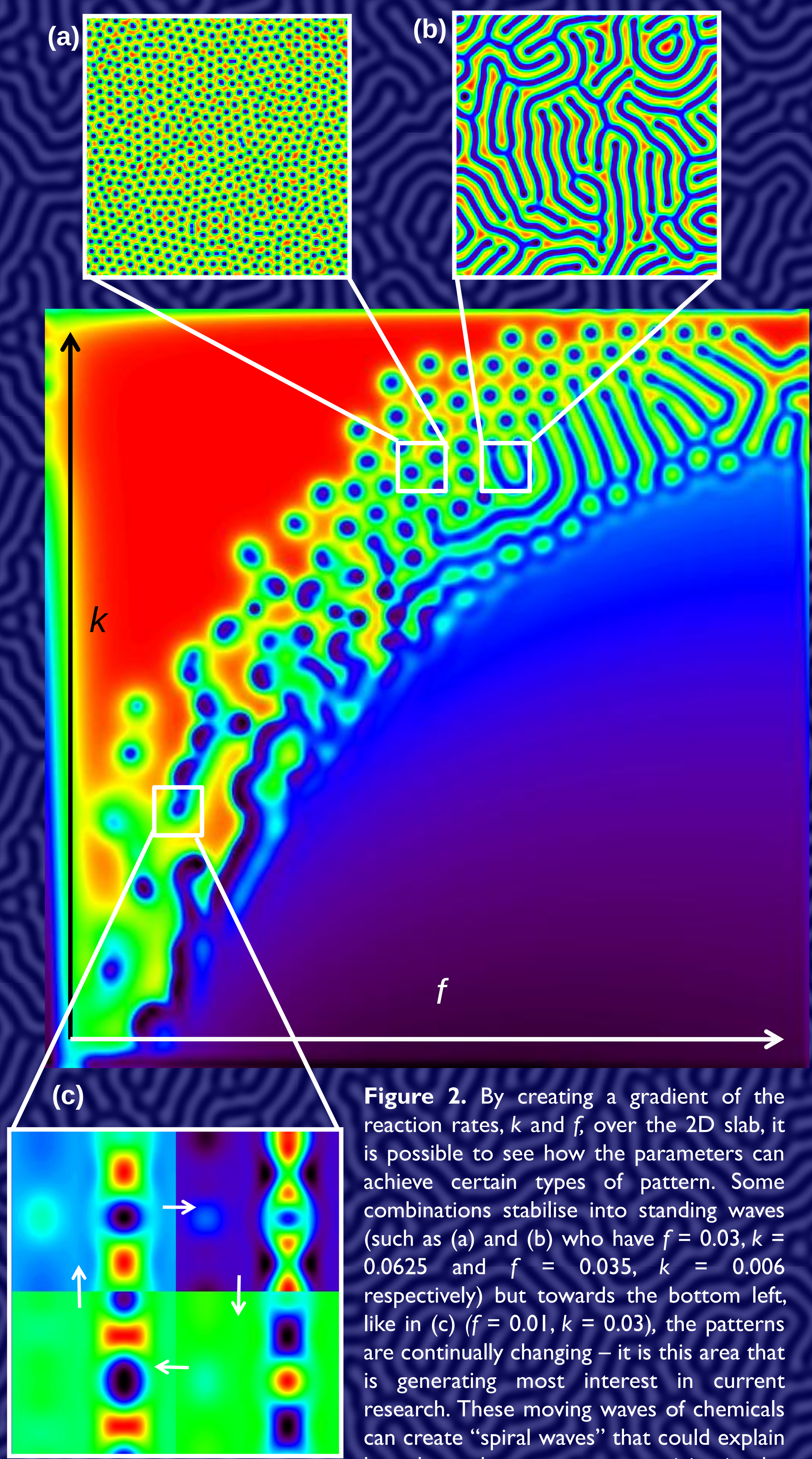

Figure 2. By creating a gradient of the reaction rates, *k* and *f*, over the 2D slab, it is possible to see how the parameters can achieve certain types of pattern. Some combinations stabilise into standing waves (such as (a) and (b) who have f = 0.03, k = 0.0625 and f = 0.035, k = 0.006 respectively) but towards the bottom left, like in (c) (f = 0.01, k = 0.03), the patterns are continually changing – it is this area that is generating most interest in current research. These moving waves of chemicals can create "spiral waves" that could explain how heart beats occur, or activity in the brain.

3. Analysing the Patterns

The patterns can be quantified by finding the average "wavelength" of the spots or stripes. This is made easier by calculating the Fourier transform of the pattern and considering it in *k*-space (see Fig. 3).

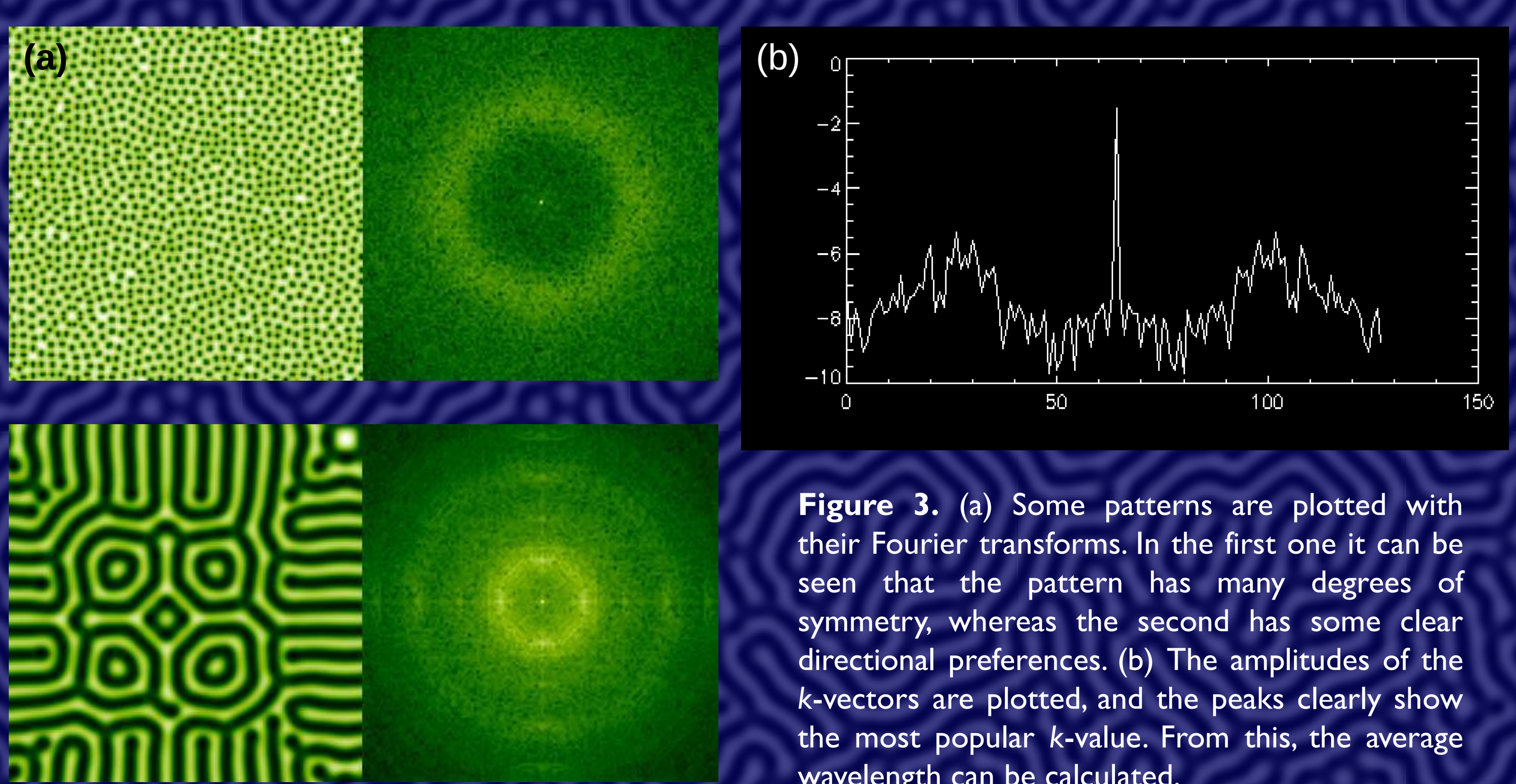

Figure 3. (a) Some patterns are plotted with their Fourier transforms. In the first one it can be seen that the pattern has many degrees of symmetry, whereas the second has some clear directional preferences. (b) The amplitudes of the *k*-vectors are plotted, and the peaks clearly show the most popular *k*-value. From this, the average wavelength can be calculated.

By finding the most common *k*-values, the average wavelength of the pattern can be found using

$$\lambda = \frac{2\pi}{k} \qquad (4)$$

For example, the first pattern in Fig. 3 (a) can be calculated to have an average wavelength of 3×10^{-4}m. Further work in this project would focus towards finding a relationship between the average wavelength of the pattern, and the combination of the variables *f* and *k*.

4. Applying Reaction-Diffusion Systems to Nature

Research into this area has shown that more complicated reaction-diffusion models can simulate patterns that are found naturally in the world – most strikingly, those on some animal pelts. By using the correct parameters, tiger/zebra stripes, leopard spots, and patterns found on snakes, cows and fish can all be reproduced (see Fig.4). There are many areas that are currently poorly understood that could potentially be explained with reaction-diffusion systems, such as activity in the brain and heart, predator-prey relationships etc., but at the moment there is only circumstantial evidence to suggest the connections, in the form of simulations such as those produced in this project.

Figure 4. These diagrams[iii][iv] show how reaction-diffusion systems may recreate patterns that are found in nature. More complicated systems recreate the patterns much more accurately.

References:
[1] A. M. Turing, *The Chemical Basis of Morphogenesis*, Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, **237**, No. 641, pp. 37-72, (1952)
[2] J.E. Pearson, *Complex patterns in a simple system*, Science **261**, pp. 189-192, (1993)
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[i] *http://psychology.wikia.com/wiki/Alan_Turing*
[ii] *http://www.pnas.org/content/103/43/15727/F1.expansion.html*
[iii] *http://www.sodahead.com/living/can-a-leopard-change-hisher-spots/question-898307/*
[iv] *http://www.tigerproperty.co.uk/*